Motor Parameters Application Note

By Jack Marsh

Introduction

Linear motor parameters can be confusing and difficult to understand, especially when different manufacturers use different methods to formulate and present them. This application note will start with first principles and establish motor parameter formulations that are consistent and convenient to measure. It will also give the relationships between certain motor parameters, show how some values differ depending on the winding type of the motor, and show how some parameters are not dependent on the winding type. These formulas and relationships will then be compared with experimentally measured values to gauge their accuracy.

Force Constant

Both the electric field and magnetic field can be defined from the Lorentz force law:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\mathbf{x}\vec{\mathbf{B}}$$

This can be simplified to calculate the force on current carrying conductors in a magnetic field as

(1) $\quad \mathbf{F} = i\mathbf{L} \times \mathbf{B}$.

In three phase linear motors where **L** and **B** are normal vectors, this reduces to

(2) $\quad F = iLB = L\sum_{n=1}^{3} i_{pn} B_{pn}$,

where F is the total force produced by the motor, L is the length of conductor in the flux field for each phase, i_p is the phase current, and B_p is the magnetic flux through each phase. The flux is described by

(3) $\quad B_p = B\sin(\theta)$,

where θ is the magnetic phase angle and B is the maximum magnetic flux. Using sinusoidal commutation results in sinusoidal currents in each phase,

(4) $\quad i_p = I_{phase}\sin(\theta),$

where I_{phase} is the peak current passing through each motor phase. Substituting equation (3) and (4) into (2) and assuming 120° phase spacing yields:

(5) $\quad F = L(I_{phase}B\sin^2(\theta) + I_{phase}B\sin^2(\theta+120) + I_{phase}B\sin^2(\theta-120))$.

Expanding and reducing using trigonometric identities yields:

$$F = LBI_{phase}(\sin^2(\theta) + \sin^2(\theta + 120) + \sin^2(\theta - 120))$$

$$F = LBI_{phase}(\sin^2\theta + (\sin\theta\cos120 + \sin120\cos\theta)^2 + (\sin\theta\cos(-120) + \sin(-120)\cos\theta)^2)$$

$$F = LBI_{phase}(\sin^2\theta + (\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta)^2 + (-\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta)^2)$$

$$F = LBI_{phase}(\sin^2\theta + \frac{3}{4}\cos^2\theta - \frac{\sqrt{3}}{2}\sin\theta\cos\theta + \frac{1}{4}\sin^2\theta + \frac{3}{4}\cos^2\theta + \frac{\sqrt{3}}{2}\sin\theta\cos\theta + \frac{1}{4}\sin^2\theta)$$

(6)

$$F = LBI_{phase}(\frac{3}{2}\sin^2\theta + \frac{3}{2}\cos^2\theta) = \frac{3}{2}LBI_{phase}(\sin^2\theta + \cos^2\theta)$$

$$F = \frac{3}{2}LBI_{phase}.$$

This can be used to define the motor's force constant, K_f, as

(7) $$K_f = \frac{F}{I_{phase}} = \frac{3}{2}LB.$$

When L is measured in meters and B is measured in Tesla (10,000 Gauss), K_f has units of N/A.

In the case of a Delta wound motor, see Figure 1, the amplitude of the current waveform in the motor lead is different than the amplitude of the current waveform in the motor phase.
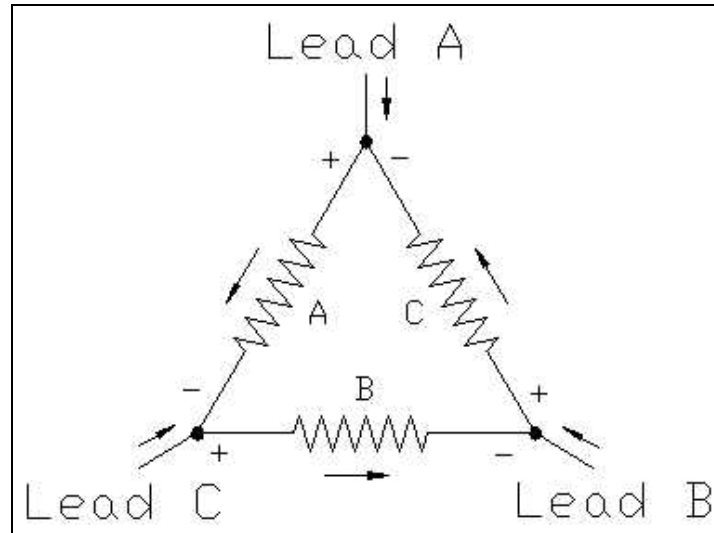

Figure 1. Delta Wound Motor

Since the motor lead current waveform can be measured more easily, it is more convenient to express the force constant in terms of motor lead amplitude, I_{lead}. The relationship between them can be determined by evaluating

$$i_{LA-\Delta} = i_{pA} - i_{pC} = I_{phase}\sin(\theta) - I_{phase}\sin(\theta + 120)$$

Reducing yields,

$$i_{LA-\Delta} = I_{phase}(\sin(\theta) - \sin(\theta)\cos(120) - \cos(\theta)\sin(120))$$

$$i_{LA-\Delta} = I_{phase}(\sin(\theta) + \frac{\sin(\theta)}{2} - \frac{\sqrt{3}}{2}\cos(\theta))$$

$$i_{LA-\Delta} = \sqrt{3}I_{phase}(\frac{\sqrt{3}}{2}\sin(\theta) - \frac{1}{2}\cos(\theta))$$

$$i_{LA-\Delta} = \sqrt{3}I_{phase}(\cos(30)\sin(\theta) - \sin(30)\cos(\theta))$$

(8) $i_{LA-\Delta} = \sqrt{3}I_{phase}\sin(\theta - 30)$.

This shows that the motor lead current waveform in a Delta wound motor is phase shifted 30 degrees and has larger amplitude than the motor phase current waveform. Specifically,

(9) $I_{Lead-\Delta} = \sqrt{3}I_{phase}$.

Using (9) and (7), a practical force constant for a Delta wound motor can be defined as

(10) $K_{f-\Delta} = \dfrac{F}{I_{Lead-\Delta}} = \dfrac{\sqrt{3}}{2}LB$.

Since the motor lead current waveform is the same as the motor phase current waveform in a WYE wound motor (see Figure 2), the force constant definition does not need to be adjusted.

(11) $K_{f-Y} = \dfrac{F}{I_{Lead-Y}} = \dfrac{F}{I_{phase}} = \dfrac{3}{2}LB$.
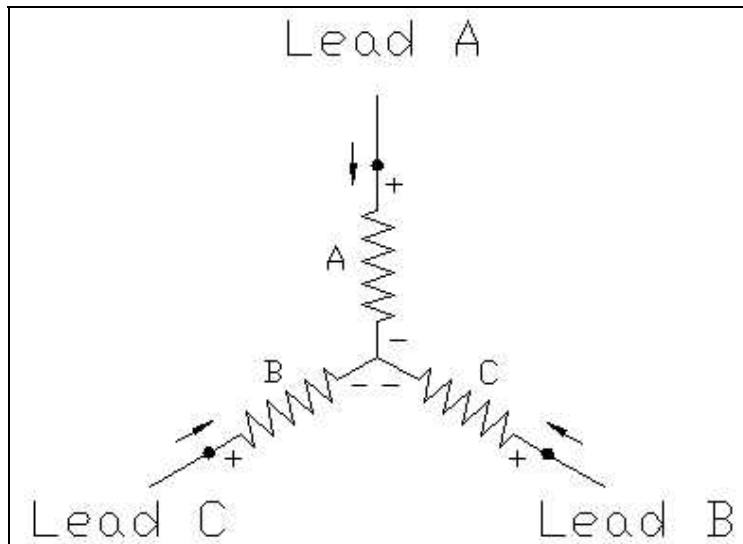

Figure 2. WYE wound motor

BEMF Constant

Faraday's Law can be used to define the motional emf voltage of a conductor in a magnetic field as:

(12) $\quad \mathrm{emf} = \mathrm{vBL}$.

This can be used to define the motor's BEMF constant, K_e, as the amplitude of the voltage waveform generated by a motor phase as it is moved through the magnetic field at a constant velocity divided by that velocity. When L is measured in meters and B is measured in Tesla, K_e has units of V/(m/s).

(13) $\quad K_{e-phase} = \dfrac{\left|emf\right|}{v} = LB$.

It is more convenient to state K_e in terms of motor lead voltage since this is generally easier to measure than phase voltage. This leads to different expressions of K_e for Delta and WYE wound motors. In the case of a Delta wound motor, the motor phase voltage is the same as the motor lead voltage, so

(14) $\quad K_{e-\Delta} = LB$.

The motor lead voltage of a WYE wound motor will be the difference of two motor phase voltages.

(15) $\quad emf_{lead-Y} = vL(B\sin(\theta) - B\sin(\theta - 120))$

Reducing yields;

$$emf_{lead-Y} = vLB(\sin(\theta) - \sin(\theta)\cos(120) + \cos(\theta)\sin(120))$$

$$emf_{lead-Y} = vLB(\frac{3}{2}\sin(\theta) + \frac{\sqrt{3}}{2}\cos(\theta))$$

$$emf_{lead-Y} = \sqrt{3}vLB(\cos(30)\sin(\theta) + \sin(30)\cos(\theta))$$

(16) $\quad emf_{lead-Y} = \sqrt{3}vLB\sin(\theta + 30)$

This shows that the voltage waveform measured between two motor leads of a WYE connected motor will have larger amplitude and be shifted 30 degrees compared to the motor phase voltage waveform. Using (13) and (16), a practical definition for K_{e-Y} can be created,

(17) $\quad K_{e-Y} = \dfrac{\left|emf_{lead-Y}\right|}{v} = \sqrt{3}LB$.

Note that the ratio between the force constant and BEMF constant is not dependant on the winding type;

(18) $\quad \dfrac{K_{f-Y}}{K_{e-Y}} = \dfrac{K_{f-\Delta}}{K_{e-\Delta}} = \dfrac{\sqrt{3}}{2}$.

Instantaneous Motor Power

The total power required by the motor can be expressed as,

(19) $$P_{total} = \sum_{n=1}^{3} i_{pn} V_{pn} \ .$$

The voltage required for each phase is given by,

(20) $$V_p = i_p R_{phase} + emf = R_{phase} I_{phase} \sin(\theta) + vBL\sin(\theta) = (R_{phase} I_{phase} + vBL)\sin(\theta) \ .$$

Substituting (20) into (19) yields

$$P_{total} = (I_{phase}^2 R_{phase} + I_{phase} vBL)(\sin^2(\theta) + \sin^2(\theta + 120) + \sin^2(\theta - 120))$$

(21) $$P_{total} = \frac{3}{2} I_{phase}^2 R_{phase} + \frac{3}{2} vBLI_{phase}$$

Using (6), this can be written as

(22) $$P_{total} = \frac{3}{2} I_{phase}^2 R_{phase} + vF$$

The relationships between lead-to-lead resistance and phase resistance for Delta and WYE wound motors are as follows,

(23) $$R_{l-l-\Delta} = \frac{2}{3} R_{phase}$$

(24) $$R_{l-l-Y} = 2R_{phase}$$

Using these relationships and (9), (22) can be expressed as

(25)
$$P_{total} = \frac{3}{2}(\frac{3}{2} R_{l-l-\Delta})(\frac{I_{Lead-\Delta}}{\sqrt{3}})^2 + vF = \frac{3}{4} R_{l-l-\Delta} I_{Lead-\Delta}^2 + vF$$

$$P_{total} = \frac{3}{2}(\frac{1}{2} R_{l-l-Y}) I_{Lead-Y}^2 + vF = \frac{3}{4} R_{l-l-Y} I_{Lead-Y}^2 + vF$$

Note that when the power is expressed in terms of lead-to-lead resistance and motor lead current waveform amplitude, the expressions for Delta and WYE wound motors are identical.

The first term in (25) is the thermal power and the second term is the mechanical power. From the standpoint of motor sizing, thermal power is the critical parameter since motor performance is limited by the temperature rise of the windings. However, total power must be considered when sizing the amplifier and power supply.

Care must be taken when calculating the average power required for a complete motion profile. The electrical energy converted into kinetic energy during acceleration is usually returned to the system during deceleration. Energy expended as mechanical work done while moving, such as metal cutting, is not returned and is lost in some other way. Attention must be paid to the sign of F in the mechanical power term to make sure that the total power is calculated correctly.

Example: **Constant Acceleration**
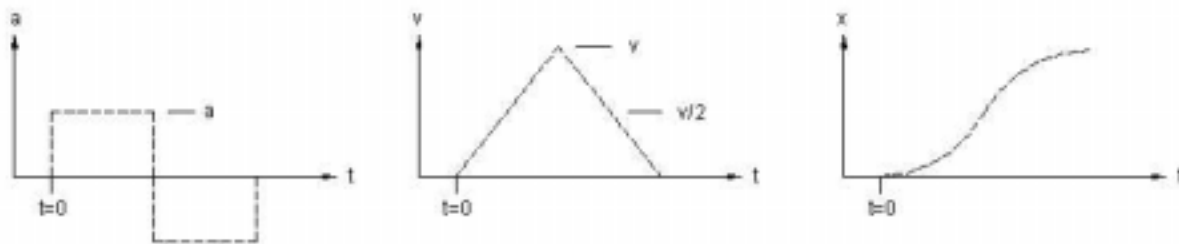
The conversion of electrical energy to kinetic energy is simple to examine when acceleration is constant and frictional losses are neglected. Consider accelerating a mass, m, at a constant rate, a, to final velocity, v, in time t. The force required to accelerate this mass is F, which equals ma. The energy required is the average power required times t. The average power required is the average velocity, v/2, times F. So the electrical energy required is

$$E = \frac{v}{2}Ft = \frac{v}{2}mat = \frac{v}{2}m\frac{v}{t}t = \frac{1}{2}mv^2 \; .$$

Notice that the electrical energy calculated above is exactly equal to the kinetic energy of the mass m moving at velocity v.

When decelerating back to zero velocity, F will equal –ma. The kinetic energy will be converted back into electrical energy and returned to the system bus.

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Motor Constant

The motor constant, K_m, is a figure of merit used to compare the relative efficiencies of different motors. It is expressed as an amount of force produced divided by the square root of the power dissipated while producing that force,

$$(26) \qquad K_m = \frac{F}{\sqrt{P_{thermal}}} \, .$$

A higher value of K_m means the motor can produce more force for a given amount of power lost. It is convenient to expand this and express K_m in terms of K_f and the lead-to-lead resistance as follows,

$$(27) \qquad K_m = \frac{F}{\sqrt{\frac{3}{4} R_{l-l} I_{lead}^2}} = \frac{2}{\sqrt{3}} \frac{K_{f-\Delta}}{\sqrt{R_{l-l-\Delta}}} = \frac{2}{\sqrt{3}} \frac{K_{f-Y}}{\sqrt{R_{l-l-Y}}}$$

As long as lead-to-lead resistance and lead current amplitude are used, K_m does not depend on the winding type.

Since the motor resistance increases with temperature, the motor constant will decrease with temperature. Most motors experience some temperature increase during operation, so it is useful to know the motor constant as a function of temperature. The resistance of copper increase 0.393% per degree C temperature rise, which leads to the following formula for the hot K_m,

$$(28) \qquad K_{m-hot} = \frac{2}{\sqrt{3}} \frac{K_{f-lead}}{\sqrt{(1 + \frac{0.393(T_{hot} - T_{cold})}{100}) R_{l-l-cold}}} \, .$$

Where T_{hot} is the operating temperature of the motor, T_{cold} is the temperature at which $R_{l-l-cold}$ is reported (usually 20 or 25 deg C), and K_{f-lead} is the value for the force constant given when using the motor lead current waveform amplitude.

Experimental Result

Measurements of BEMF, force, current, voltage and power were taken using a Trilogy 310-6 ironless linear motor. The motor applied a static force to an Omega LC203-200 load cell. The voltage, current, and power were measured using a power meter developed using Cirrus Logic CS5460 power meter chips. The motor was controlled by a Delta Tau PMAC controller sinusoidally commutating a Trust Automation TA320 linear amplifier operating on a 120 VDC bus. Figure 3 shows the equipment used.

Figure 3. Force and Power Test Equipment

The motor's BEMF constant was determined by moving the motor by hand and using an oscilloscope to measure the resulting voltage waveform. Figure 4 shows the waveforms.

The magnetic cycle length of the Trilogy 310 series motor is 2.4" or 60.96mm. The peak-to-peak voltage and period of the BEMF waveform for two phases of the motor are given by the oscilloscope. These values can be used to calculate K_e. This motor is normally connected in Delta.

$$(29) \qquad K_e = \frac{BEMF_{0-pk}}{Velocity} = \frac{BEMF_{pk-pk}/2}{Velocity} = \frac{BEMF_{pk-pk}/2}{CycleLength/period} = \frac{(BEMF_{pk-pk})(period)}{2(CycleLength)}$$

$$K_{e-\Delta}(CH1) = \frac{(386V)(0.03176\sec)}{2(0.06096m)} = 100.55\frac{V}{m/\sec}$$

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$$K_{e-\Delta}(CH2) = \frac{(390V)(0.03156\sec)}{2(0.06096m)} = 100.95\frac{V}{m/\sec}$$

$$K_{e-\Delta}(avg) = 100.8\frac{V}{m/\sec}$$

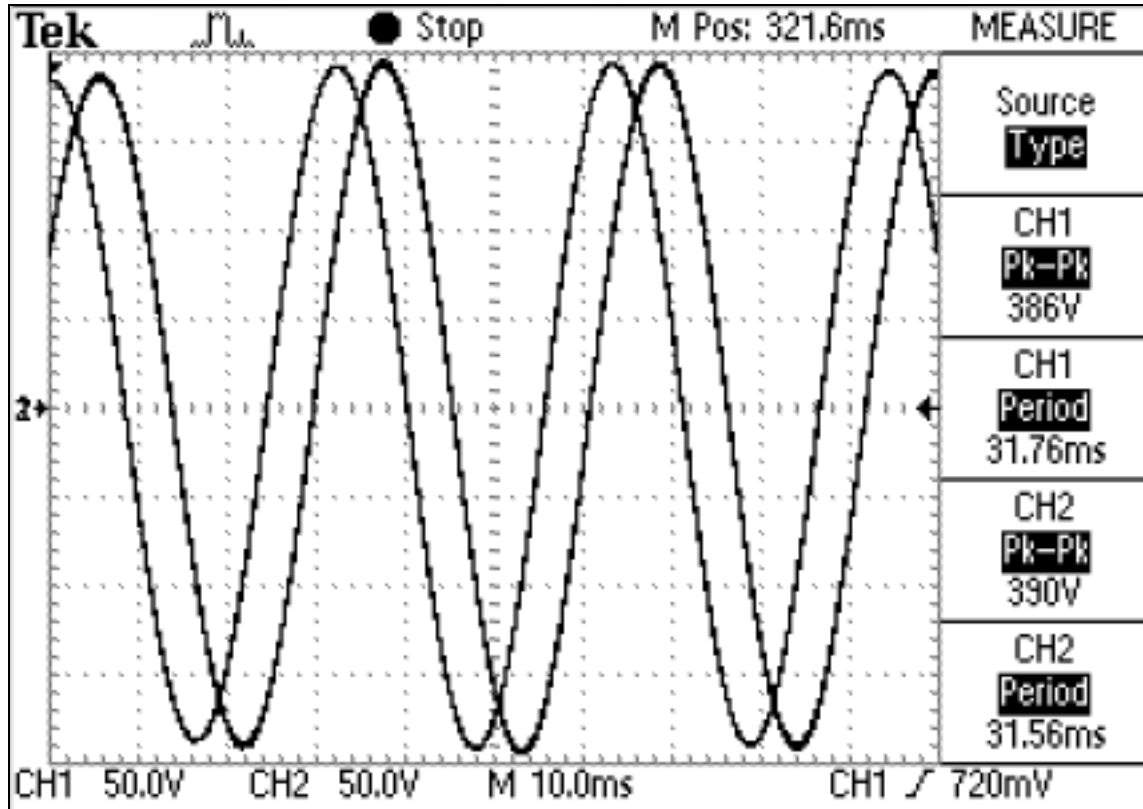**Figure 4. Trilogy 310-6 BEMF Waveform**

The lead to lead motor resistance was measured using an Ohmmeter,

$$R_{l-l-\Delta} = 24.4 Ohms$$

The force measured by the load cell was 78 pounds,

$$F = 78\,pounds = 347N$$

The motor lead current waveform amplitude, motor lead voltage waveform amplitude, energy, and power were written to a text file every ¼ second by the power meter. This data is shown in Figure 5. Columns 2, 3, and 4 show the RMS current, RMS voltage, and energy respectively for that ¼ second period. Column 5 shows the RMS value of all the steps to that point. Column 6 shows the total energy to that point. Column 7 shows the average power up to that point (E_{total} / Total time). Column 8 shows K_m calculated using the force produced divided by the square root of the power.

Measured Data							Calculated Data
Time	Irms	Vrms	E	Irms Total	E Total	Pavg	Motor Constant -
[s]	[A]	[V]	[J]	[A]	[J]	[W]	F/sqrtP [N/sqrtW]
0.75	4.0	82.2	71.3	4.0	214.6	286.1	20.5
1.00	4.0	83.0	71.4	4.0	286.0	285.9	20.5
1.25	4.0	82.6	71.5	4.0	357.5	285.9	20.5
1.50	4.0	82.9	71.2	4.0	428.7	285.7	20.5
1.75	4.0	82.6	71.2	4.0	499.9	285.6	20.5
2.00	4.0	82.2	71.2	4.0	571.1	285.5	20.5
2.25	4.0	82.7	71.2	4.0	642.3	285.4	20.5
2.50	4.0	82.3	71.2	4.0	713.5	285.3	20.5
2.75	4.0	82.7	71.2	4.0	784.6	285.2	20.6
3.00	4.0	82.4	71.1	4.0	855.8	285.2	20.6
3.25	4.0	82.7	71.1	4.0	926.9	285.1	20.6

Figure 5. Output of the Power Meter

The power meter used measures the peaks of the voltage and current waveforms and the energy several times every millisecond. It then calculates and reports RMS values for the measured **peak-of-sine-wave** current and voltage values and reports those for every user selected time interval, in this case 0.25 seconds. It also calculates an RMS value for all of the **peak-of-sine-wave** current measurements and reports the new total RMS current value every time interval.

> ***Care must be taken when using equipment to measure motor voltage and/or current values. There are many different ways to report these values (for example, peak of sine or RMS of sine) and the results can be very confusing or misleading if the correct formulation is not known. The relationships derived so far use peak of sine (that is, amplitude) values. The equipment manufacturer should be able to clarify how the values are formulated and how they relate to the peak of sine values.***

In this case the motor was stationary and putting out a constant force, so the voltage and current were not changing and the RMS values are the same as the peak values.

The force constant can be calculated two different ways; by using F/I or by using the relationship between K_e and K_f given by (18). (Note that these values agree to within about 0.6%)

$$K_{f-\Delta} = \frac{\sqrt{3}}{2} K_{e-\Delta} = \frac{\sqrt{3}}{2}(100.8) = 87.3 \frac{N}{A}$$

$$K_{f-\Delta} = \frac{347 N}{4.0 A} = 86.8 \frac{N}{A}$$

The motor power can be calculated three different ways

- Energy divided by time as reported by the power meter,

- Equation (25) using the measured lead to lead resistance and reported lead current,

- Using the lead current and lead voltage as reported by the power meter since this was a stationary test and no electrical energy was converted to kinetic energy.

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Since the motor lead voltage waveform and motor lead current waveform are not in phase once the motor is connected in Delta or WYE (see the derivations of (8) and (16)) simply multiplying their amplitudes to obtain total power is not correct. To derive the correct formula for power as a function of peak lead voltage and peak lead current, the relationship between peak lead voltage, peak lead current, and lead to lead resistance must be determined and substituted into (25). In this special case, the motor is stationary so the motional emf can be neglected.

In the case of a Delta wound motor, peak lead voltage is the same as peak phase voltage. So,

$$(30) \qquad V_{lead-\Delta} = V_{phase} = I_{phase} R_{phase} = \left(\frac{I_{lead}}{\sqrt{3}} \right) \left(\frac{3}{2} R_{l-l} \right) = \frac{\sqrt{3}}{2} I_{lead} R_{l-l} \text{ **}$$

In the case of a WYE wound motor, the lead voltage is the difference of two phase voltages,

$$V_{l-Y} = V_{phase} \sin(\theta) - V_{phase} \sin(\theta - 120) = I_{phase} R_{phase} \sin(\theta) - I_{phase} R_{phase} \sin(\theta - 120) \text{ **}$$

This reduces to

$$V_{l-Y} = \sqrt{3} I_{phase} R_{phase} \sin(\theta + 30) \text{ **}$$

This leads to

$$(31) \qquad V_{lead-Y} = |V_{l-Y}| = \sqrt{3} I_{phase} R_{phase} = \sqrt{3} I_{lead} \left(\frac{R_{l-l}}{2} \right) = \frac{\sqrt{3}}{2} I_{lead} R_{l-l} \text{ **}$$

Note from (30) and (31) that the peak lead voltage as a function of lead to lead resistance and peak lead current does not depend on the winding type. Substituting (30) or (31) into (25) and assuming v=0,

$$(32) \qquad P_{total} = \frac{3}{4} R_{l-l} I_{lead}^2 = \frac{\sqrt{3}}{2} I_{lead} \left(\frac{\sqrt{3}}{2} I_{lead} R_{l-l} \right) = \frac{\sqrt{3}}{2} I_{lead} V_{lead} \text{ **}$$

Note: these relationships are only valid when the motor is stationary. They do not include emf.

The motor power calculated using (32) and an average value of the reported voltage is

$$P = \frac{\sqrt{3}}{2} (4.0A)(82.6V) = 286.1W .$$

The motor power calculated using (25) is

$$P = \frac{3}{4} (4.0A)^2 (24.4 Ohms) = 292.8W .$$

The average motor power reported by the power meter is

$$P = 285.5W .$$

These values agree to within about 2.5%.

The motor constant calculated by using (26), the power meter output, and the force reported by the load cell is

$$K_m = \frac{F}{\sqrt{P_{thermal}}} = \frac{347N}{\sqrt{285.5W}} = 20.5\frac{N}{\sqrt{W}} \ .$$

The motor constant calculated by using (27) is

$$K_m = \frac{2}{\sqrt{3}}\frac{K_{f-\Delta}}{\sqrt{R_{l-l-\Delta}}} = \frac{2}{\sqrt{3}}\frac{87.1\,N/A}{\sqrt{24.4Ohms}} = 20.4\frac{N}{\sqrt{W}} \ .$$

These values agree to within about 0.5%.

Summary

The Lorentz force law can be used to express the motor force constant, K_f, as a function of the total length of wire in one phase that passes through the magnetic field, L, and the maximum strength of the magnetic field, B. This value depends on the winding type;

(10) $\qquad K_{f-\Delta} = \frac{\sqrt{3}}{2}LB$

(11) $\qquad K_{f-Y} = \frac{3}{2}LB$

Faraday's Law can be used to express the motor BEMF constant, K_e, as a function of LB as well. This value also depends on the winding type,

(14) $\qquad K_{e-\Delta} = LB$

(17) $\qquad K_{e-Y} = \sqrt{3}LB$

The ratio of force constant to BEMF constant is independent of the winding type;

(18) $\qquad \dfrac{K_{f-Y}}{K_{e-Y}} = \dfrac{K_{f-\Delta}}{K_{e-\Delta}} = \dfrac{\sqrt{3}}{2} \ .$

The instantaneous motor power can be determined by summing the power of the three phases. As long as lead to lead resistance and peak motor lead current are used, this value is independent of the winding type.

(25) $\qquad P_{total} = \frac{3}{4}R_{l-l}I^2_{Lead} + vF$

The first term is the thermal power dissipated in the motor and the second term is the mechanical power. Care must be used when calculating the average power for a complete motion profile to make sure the sign of the mechanical power term is considered.

When the motor is stationary there is no mechanical power, so the power can be calculated using

$$(32) \qquad P_{total} = \frac{\sqrt{3}}{2} I_{lead} V_{lead} \quad \textbf{**stationary motor only}$$

The motor constant, K_m, is a figure of merit used to compare the relative efficiencies of different motors. It is defined as the amount of force produced divided by the square root of the thermal power dissipated while producing that force. Mechanical power is not included in the definition of K_m because mechanical power is a function of velocity and including it would make K_m a function of velocity as well, not a constant. K_m does not depend on the winding type as long as lead to lead resistance and peak lead current are used.

$$(27) \qquad K_m = \frac{2}{\sqrt{3}} \frac{K_{f-\Delta}}{\sqrt{R_{l-l-\Delta}}} = \frac{2}{\sqrt{3}} \frac{K_{f-Y}}{\sqrt{R_{l-l-Y}}}$$

Since the motor resistance is a function of temperature and the motor experiences some temperature rise in most applications, it is useful to know the formula for K_m as a function of temperature.

$$(28) \qquad K_{m-hot} = \frac{2}{\sqrt{3}} \frac{K_{f-lead}}{\sqrt{(1 + \frac{0.393(T_{hot} - T_{cold})}{100})R_{l-l-cold}}}$$

The formulas and relationships given here correlate extremely well with experimental results.